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SHORT COMMUNICATIONS

A REPLACEMENT FOR THE SRSS METHOD IN SEISMIC ANALYSIS

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SUMMARY

It is well-known that the application of the Square-Root-of-Sum-of-Squares (SRSS) method in seismic analysis for combining modal maxima can cause significant errors. Nevertheless, this method continues to be used by the profession for significant buildings. The purpose of this note is to present an improved technique to be used in place of the SRSS method in seismic analysis.

A Complete Quadratic Combination (CQC) method is proposed which reduces errors in modal combination in all examples studied. The CQC method degenerates into the SRSS method for systems with well-spaced natural frequencies. Since the CQC method only involves a small increase in numerical effort, it is recommended that the new approach be used as a replacement for the SRSS method in all response spectrum calculations.

INTRODUCTION

The SRSS method of combining modal maxima has found wide acceptance among structural engineers engaged in seismic analysis. For most two-dimensional analyses, the SRSS method appears to yield good results when compared to time-history response calculations. Based upon the early success of the method in two-dimensions, the SRSS approach is now being used for three-dimensional dynamic analysis without having been verified for such structures. In fact, the method is now an integral part of a large number of computer programs for the dynamic analysis of general three-dimensional systems.[1-3]

The problem associated with the application of the SRSS method can be illustrated by its application to the four-storey building shown in Figure 1. The building is symmetrical; however, the centre of mass is located 25 inches from the geometric centre of the building. The direction of the applied earthquake motion, a table of natural frequencies and the principal directions of the mode shapes are illustrated in Figure 2. One notes the closeness of the frequencies and the complex nature of the mode shapes in which the fundamental mode shape

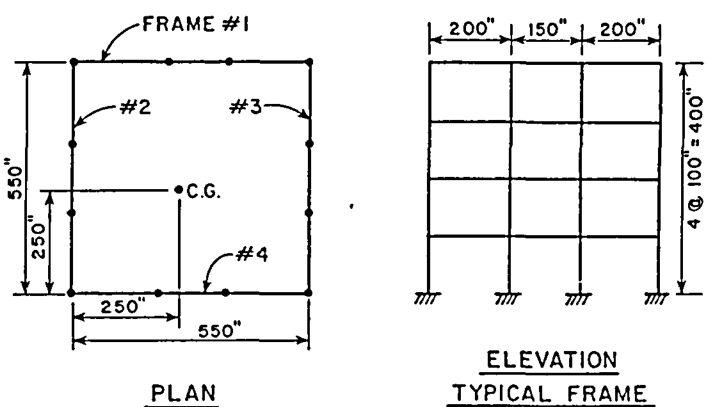

Figure 1. Simple three-dimensional building example

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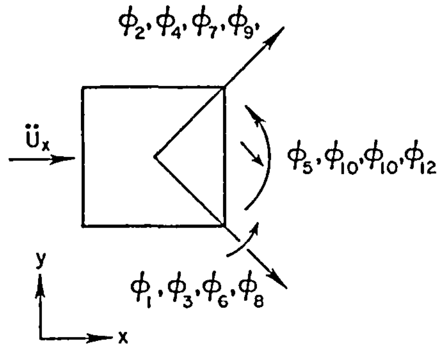

MODE	FREQUENCIES (RADIANS/SEC.)
1	13.869
2	13.931
3	43.995
4	44.189
5	54.418
6	77.686
7	78.029
8	108.32
9	108.80
10	172.613
11	304.80
12	425.00

Figure 2. Frequencies and approximate directions of mode shapes

has x, y, as well as torsional components. This type of frequency distribution and coupled mode shapes are very common in asymmetrical building systems.

This structure was subjected to the Taft, 1952, earthquake. The exact maximum base shears for the four exterior frames produced in the first five modes are shown in Figure 3. A mode superposition solution, in which all 12 modes were used, produces base shears as a function of time. The maximum resulting base shears in each of the four frames are plotted in Figure 4(a). For this structural model and loads, these base shears represent the 'exact' results.

If these modal base shears are combined by the SRSS method, the values shown in Figure 4(b) are obtained. The sum of the absolute values of the base shears is shown in Figure 4(c). The base shears found by the new 'Complete Quadratic Combination' (CQC) method are shown in Figure 4(d). Note that the signs of the base shears are not retained in any of these approximate methods.

For this case it is clear that the SRSS method greatly underestimates the forces in the direction of the motion. Also, the base shears in the frames normal to the motion are overestimated by a factor of 14. It is clear that errors of this order are not acceptable. The sum of absolute values, which is a method normally suggested for the case where frequencies are close, gives a good approximation of the forces in the direction of motion but overestimates the forces in the normal frames by a factor of 25. For this example, the double sum method required by the Nuclear Regulatory Commission produces results very close to the sum of the abolute values.

The CQC method applied to this example gives an excellent approximation to the exact results. The main purpose of this note is to present a summary of this new technique for combining model maxima.

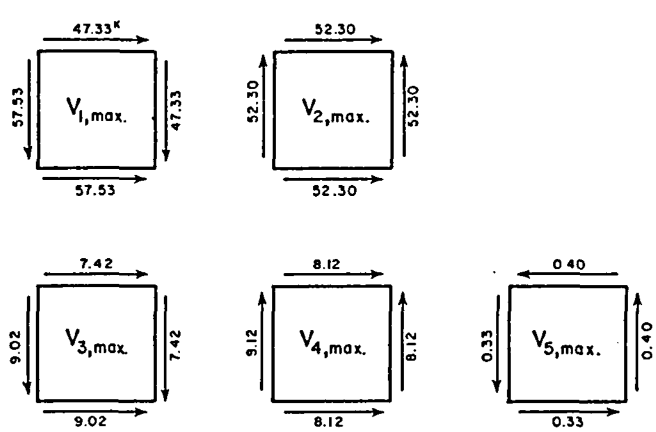

Figure 3. Base shears in first five modes

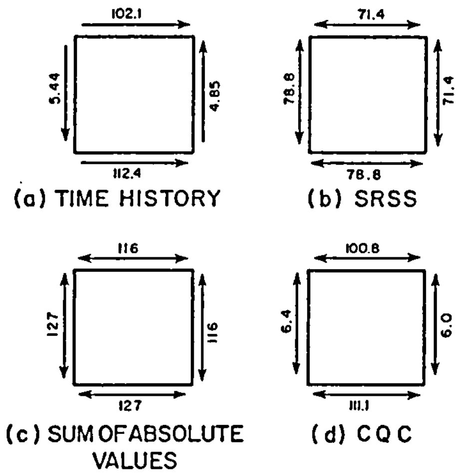

Figure 4. Comparison of modal combination methods

BASIC MODAL EQUATIONS

The dynamic equilibrium equations for a three-dimensional structural system subjected to a ground acceleration, $\ddot{u}_x(t)$, in the x-direction is written as

$$\mathbf{M}\ddot{\mathbf{U}}+\mathbf{C}\dot{\mathbf{U}}+\mathbf{K}\mathbf{U} = \mathbf{M}_x\langle \ddot{u}_x(t)\rangle \tag{1}$$

where **M**, **C** and **K** are the mass, damping and stiffness matrices, respectivley. The three-dimensional relative displacements, velocities and accelerations are indicated by $\mathbf{U}$, $\dot{\mathbf{U}}$ and $\ddot{\mathbf{U}}$. The column vector $\mathbf{M}_x$ contains the components of mass in the x-direction and zeros in all other directions.

The mode superposition solution involves the introduction of the transformation

$$\mathbf{U} = \mathbf{\Phi}\mathbf{Y} \tag{2}$$

where $\mathbf{\Phi}$ is the matrix containing three-dimensional mode shapes of the system and $\mathbf{Y}$ is the vector of normal co-ordinates. The introduction of this transformation and the premultiplication of equation (1) by $\mathbf{\Phi}^{\mathrm{T}}$ yields

$$\mathbf{\Phi}^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}\ddot{\mathbf{Y}}+\mathbf{\Phi}^{\mathrm{T}}\mathbf{C}\mathbf{\Phi}\dot{\mathbf{Y}}+\mathbf{\Phi}^{\mathrm{T}}\mathbf{K}\mathbf{\Phi}\mathbf{Y} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{M}_x\langle \ddot{u}_x(t)\rangle \tag{3}$$

For proportional damping the mode shapes have the following properties:

$$\boldsymbol{\phi}_i^{\mathrm{T}}\mathbf{M}\boldsymbol{\phi}_i = m_i \tag{4}$$

$$\boldsymbol{\phi}_i^{\mathrm{T}}\mathbf{K}\boldsymbol{\phi}_i = \omega_i^2 m_i \tag{5}$$

$$\boldsymbol{\phi}_i^{\mathrm{T}}\mathbf{C}\boldsymbol{\phi}_i = 2\zeta_i\omega_i m_i \tag{6}$$

in which $\boldsymbol{\phi}_i$ is the ith column of $\mathbf{\Phi}$ representing the ith mode shape, m_i is the ith modal mass, and ζ_i is the damping ratio for mode i. Due to the orthogonality properties of the mode shapes, all modal coupling terms of the form $\boldsymbol{\phi}_i^{\mathrm{T}}\mathbf{A}\boldsymbol{\phi}_j$ are zero for $i \neq j$. Thus, equation (3) reduces to a set of uncoupled equations in which the typical 'modal' equation is of the form:

$$\ddot{Y}_i+2\zeta_i\omega_i\dot{Y}_i+\omega_i^2 Y_i = p_i\langle \ddot{u}_x(t)\rangle \tag{7}$$

where

$$p_i = \frac{\boldsymbol{\phi}_i^{\mathrm{T}}\mathbf{M}_x}{m_i} \tag{8}$$

is the participation factor for mode i.

The evaluation of equation (7) for all modes yields the time-history solution for normal co-ordinates. The total structural displacements, as a function of time, are then obtained from equation (2).

DEFINITION OF RESPONSE SPECTRUM AND MAXIMUM MODAL DISPLACEMENT

The following equation can be solved for the response $y_i(t)$

$$\ddot{y}_i + 2\zeta_i \omega_i \dot{y}_i + \omega_i^2 y_i = \ddot{u}_x(t) \tag{9}$$

At the point in time where $|y_i(t)|$ is maximum, the response is defined as $y_{i,\max}$. A plot of this maximum displacement versus the frequency ω_i for each ζ_i is by definition the displacement response spectrum for the earthquake $\ddot{u}_x(t)$. A plot of $y_{i,\max}\,\omega_i$ is the pseudo-velocity spectrum and a plot of $y_{i,\max}\,\omega_i^2$ is the pseudo-acceleration spectrum. These pseudo-velocity and acceleration spectra are of the same physical interest but are not an essential part of a response spectrum analysis.

If the dynamic loading on the structure is specified in terms of the displacement spectrum, then the maximum response of each mode is given by

$$Y_{i,\max} = p_i\, y_{i,\max} \tag{10}$$

Therefore, the maximum contribution of mode i to the total response of the structure is

$$\mathbf{U}_{i,\max} = \boldsymbol{\phi}_i\, p_i\, y_{i,\max} \tag{11}$$

For all modes $y_{i,\max}$ is, by definition, positive. The maximum modal displacement $\mathbf{U}_{i,\max}$ is proportional to the mode shape $\boldsymbol{\phi}_i$, and the sign of the proportionality constant is given by the sign of the modal participation factor. Therefore, each maximum modal displacement has a unique sign, which is given by equation (11). Also, the maximum internal modal forces, which are consistently evaluated from the maximum modal displacements, have unique signs. These signs for the maximum modal base shears of the example structure are indicated in Figure 3 by arrows.

THE COMPLETE QUADRATIC COMBINATION METHOD (CQC)

The use of random vibration theories can eliminate the previously illustrated errors which are inherent in the absolute sum or the SRSS method. Based on this approach, several other papers have presented more realistic methods for modal combination.[4,5] The complete development of the CQC method, which is now being proposed as a direct replacement for the SRSS technique, is presented by the second author in References 6 and 7. The CQC method requires that all modal response terms be combined by the application of the following equations:

For a typical displacement component, u_k:

$$u_k = \sqrt{\left(\sum_i \sum_j u_{ki}\, \rho_{ij}\, u_{kj}\right)} \tag{12a}$$

and for a typical force component, f_k:

$$f_k = \sqrt{\left(\sum_i \sum_j f_{ki}\, \rho_{ij}\, f_{kj}\right)} \tag{12b}$$

where u_{ki} is a typical component of the modal displacement response vector, $\mathbf{U}_{i,\max}$, and f_{ki} is a typical force component which is produced by the modal displacement vector, $\mathbf{U}_{i,\max}$. Note that this combination formula is of complete quadratic form including all cross-modal terms, hence, the reason for the name Complete Quadratic Combination. It is also important to note that the cross-modal terms in equations (12) may assume positive or negative values depending on whether the corresponding modal responses have the same or opposite signs. The signs of the modal responses are, therefore, an important key to the accuracy of the CQC method.

In general the cross-modal coefficients, ρ_{ij}, are functions of the duration and frequency content of the loading and of the modal frequencies and damping ratios of the structure. If the duration of earthquake is long compared to the periods of the structure, and if the earthquake spectrum is smooth over a wide range of frequencies, then, it is possible to approximate these coefficients by[6,7]

$$\rho_{ij}=\frac{8\sqrt{(\zeta_i\zeta_j)}(\zeta_i+r\zeta_j)r^{3/2}}{(1-r^2)^2+4\zeta_i\zeta_j r(1+r^2)+4(\zeta_i^2+\zeta_j^2)r^2} \tag{13}$$

where $r=\omega_j/\omega_i$. For constant modal damping, ζ, this expression reduces to

$$\rho_{ij}=\frac{8\zeta^2(1+r)r^{3/2}}{(1-r^2)^2+4\zeta^2 r(1+r)^2} \tag{14}$$

Note that for equal damping and $r=1$, $\rho_{ij}=1$. Expressions for the cross terms, which take into account the duration and frequency content of the loading, are given in Reference 7. Additional modal combination rules giving the variability and the distribution of the peak response are also given in that reference. These rules are useful in non-deterministic analysis.

Considering 5 per cent damping and the frequencies of the example structure, the evaluation of equation (14) yields the cross-correlation coefficients given in Table I. One notes that if the frequencies are well-separated the off-diagonal terms approach zero and the CQC method approaches the SRSS method.

Table I. Modal cross-correlation coefficients

Mode	1	2	3	4	5	Freq. rad/s
1	1·000	0·998	0·006	0·006	0·004	13·87
2	0·998	1·000	0·006	0·006	0·004	13·93
3	0·006	0·006	1·000	0·998	0·180	43·99
4	0·006	0·006	0·998	1·000	0·186	44·19
5	0·004	0·004	0·180	0·186	1·000	54·42

COMPUTER PROGRAM IMPLEMENTATION

The CQC method of modal combination has been incorporated in the computer program TABS—Three-Dimensional Analysis of Building Systems.[1] This involved the addition of one subroutine for the evaluation of modal correlation factors, Equation (14), and the replacement of the SRSS by the CQC method as given by equations (12). The increase in computer execution time due to the addition of the CQC method was insignificant (less than 0·1 per cent for a typical structure). Therefore, there is no justification to continue using the potentially erroneous SRSS method. The application of the modified TABS program to several buildings has verified the validity of the CQC method. Other examples are given in Reference 7.

FINAL REMARKS

It should be pointed out that a method similar to the CQC method was first proposed by Rosenblueth and Elorduy[4] in 1969. Their method, which has a somewhat heuristic basis, has a more complicated cross-modal term involving the duration of earthquake as well as the modal frequencies and damping values. This method has unfortunately been neglected or misrepresented over the past several years. For example, the NRC Regulatory Guide[8] recommends it for structures with closely spaced modes, however, it wrongly specifies the cross-modal terms as being always positive. This will result in overly conservative response estimates in some applications. Concern that this earlier method is being misunderstood, and the fact that the CQC method is simpler and more practical, have prompted the writing of this note.

It should also be pointed out that the SRSS method gives good results for some structures subjected to two-directional seismic input, even when the modal frequencies are closely spaced. It can be shown that this is due to cancelling of the cross-modal terms corresponding to the two directions of input. This phenomenon, however, is not generally true. For example, when the two components of input are of different intensities, or when the three-dimensional structure is highly asymmetric, the cross-modal terms would still be significant and, therefore, the SRSS method will lead to erroneous results.

Based on the preceding numerical example and the above discussion, it is strongly recommended that the use of the SRSS method for seismic response analysis of structures be immediately discontinued. Continued use of the SRSS technique may dramatically overestimate the required design forces in some structural elements or it may significantly underestimate the forces in other elements. The proposed CQC method is based on fundamental theories of random vibration and consistently yields accurate results when compared to time-history analyses.

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